

The Role of Fermions in Bubble Nucleation

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Abstract

We present a study of the role of fermions in the decay of metastable states of a scalar field via bubble nucleation. We analyze both one and three-dimensional systems by using a gradient expansion for the calculation of the fermionic determinant. The results of the one-dimensional case are compared to the exact results of previous work.

I. INTRODUCTION

The study of the mechanism of decay of metastable systems via nucleation of thermally activated bubbles finds a wide range of applicability in Condensed Matter physics. In fact, the formalism developed in [1] and [2] fits naturally in the study of the dynamics of defects in one-dimensional conducting polymers [5,6]. Applications to three-dimensional systems

are also numerous, from the behaviour of binary liquid mixtures [3,4] to problems in Optics, such as atomic ionization in ultra-strong laser fields [10], and even Baryogenesis [11].

In a previous article [2], we studied a one-dimensional system of interacting fermions and bosons that started in a metastable vacuum and gradually decayed to the true one. We investigated the nucleation of bubbles of true vacuum inside the false one via thermal activation. We analyzed the stability of these bubbles and calculated the decay rate as a function of time, in the presence of a finite density of fermions, at finite temperature. The inclusion of fermions proved to have remarkable effects as it changed qualitative features of the physics of metastability [2].

The fact that we were working with a one-dimensional system allowed us to solve the problem exactly by means of inverse scattering methods. However, as we move to higher dimensions, this mathematical tool is no longer available and we are compelled to make approximations to evaluate the fermionic determinant that encodes the fermionic effects on the system. This may be accomplished by the use of a functional gradient expansion for the determinant. Throughout this work, we perform our calculations for both one and three-dimensional systems, so that we can compare the results of our approximations on the former with the exact results of [2]. In fact, our approximation is able to detect the phenomenon of “quantum stabilization” for both one and three-spatial dimensions.

The paper is organized as follows: in section II, we present the model considered; in section III, we perform the gradient expansion for the fermionic determinant; in section IV, we obtain the form of the sphaleron solution that includes fermionic effects; in section V, we include the presence of the gap and two bound states of fermions; in section VI, we study the stability of bubble-like solutions; in section VII, we comment on the methods and present our conclusions.

II. THE MODEL

The model Lagrangian has the following form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - [V(\phi) - V(\phi_2)] + \bar{\psi}_a(i\gamma^\mu\partial_\mu - \mu - g\phi)\psi_a \quad (2.1)$$

where μ is the bare mass of the fermions, g is the coupling constant, $\psi_a(x)$ is the fermion field, a denotes fermion species, $\phi(x)$ is a scalar field and ϕ_2 is a local minimum of the potential

$$V(\phi) = \frac{g^2}{2}(\phi - \phi_0)^2 \left(\phi + \phi_0 + \frac{2\mu}{g} \right)^2 + j\phi \quad (2.2)$$

where ϕ_0 is a constant and j is an external current, responsible for the asymmetry of the potential even in the purely bosonic case. (We may find a physical realization of this form of potential in the description of conducting polymers [5,6]; see Fig. 1).

We will be interested in the effects of fermions on the bosonic field. Thus, in order to construct an effective theory for bosons, we must integrate over fermions to obtain an effective action. However, integrating over fermions implies calculating the following determinant

$$S_F = -\ln[\det(i\gamma^\mu\partial_\mu - \mu - g\phi)] = -\text{tr}[\ln(i\gamma^\mu\partial_\mu - \mu - g\phi)] \quad (2.3)$$

In one spatial dimension, it is possible to perform an exact calculation of (2.3) by making use of inverse scattering methods [2,7,8]. For three spatial dimensions, we must resort to approximations.

III. GRADIENT EXPANSION FOR THE FERMION DETERMINANT

After the fermionic integration, we may rewrite the effective action as

$$S_{eff}[\phi] = \int d^\nu x \left\{ \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - [V(\phi) - V(\phi_2)] \right\} - \text{tr}[\ln(i\gamma^\mu\partial_\mu - \mu - g\phi)] \quad (3.1)$$

where $\int d^\nu x \equiv \int_0^T dt \int d^D x$. The field configuration that extremizes (3.1) must satisfy the Euler-Lagrange equation

$$\square\phi = -\frac{\partial V}{\partial\phi} - Sp \langle x | \frac{1}{i\gamma^\mu\partial_\mu - \mu - g\phi} | x \rangle \quad (3.2)$$

where Sp means trace over the spin degrees of freedom.

We may calculate the Green function that appears in (3.2) by using a functional gradient expansion [14]. This means that we will focus on the long distance (small momentum) properties of the theory. To do so, we use the identity [12,13](see Appendix A)

$$\begin{aligned}\mathcal{G}(x, x) &\equiv Sp \langle x | \frac{1}{i\gamma^\mu \partial_\mu + M(x)} | x \rangle = \\ &= Sp \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{\gamma^\mu p_\mu + M(x)} \sum_{n=0}^{\infty} (-1)^n \left(\Delta M \left(\frac{1}{i} \frac{\partial}{\partial p}, x \right) \frac{1}{\gamma^\mu p_\mu + M(x)} \right)^n\end{aligned}\quad (3.3)$$

where

$$\Delta M \left(\frac{1}{i} \frac{\partial}{\partial p}, x \right) = \partial_\mu M(x) \frac{1}{i} \frac{\partial}{\partial p_\mu} + \frac{1}{2} \partial_\mu \partial_\nu M(x) \left(\frac{1}{i} \right)^2 \frac{\partial^2}{\partial p_\mu \partial p_\nu} + \dots \quad (3.4)$$

Keeping terms up to second order in the derivatives and explicitly performing the integrals, we obtain

$$\mathcal{G}(x, x) = \alpha_\nu M^{\nu-1} + \beta_\nu M^{\nu-4} \square M \quad (3.5)$$

where α_ν and β_ν are functions of the number of space-time dimensions:

$$\alpha_\nu \equiv \frac{\pi^{-\nu/2}}{2^{\nu+1}} \Gamma(1 - \nu/2) \quad (3.6)$$

$$\beta_\nu \equiv \frac{\pi^{-\nu/2}}{2^{\nu-1} \Gamma(\nu/2)} \left[\frac{\Gamma(\nu/2)}{4!} \Gamma(4 - \nu/2) - \frac{\Gamma(1 + \nu/2)}{2!2!} \Gamma(3 - \nu/2) + \frac{\Gamma(2 + \nu/2)}{4!} \Gamma(2 - \nu/2) \right] \quad (3.7)$$

For $\nu = 2$ and $\nu = 4$, these functions are divergent. Nevertheless, we may absorb these divergences by a suitable redefinition of the free parameters of the model. After a redefinition of g , μ and j in (2.2) and remembering that, in our case, $M(x) = -(\mu + g\phi)$, we may explicitly calculate the equation of motion (3.2).

For $\nu = 2$, equation (3.2) reads (up to order g^2)

$$\square \phi = -\frac{\partial V}{\partial \phi} \left[\frac{(\mu + g\phi)^2}{(\mu + g\phi)^2 + \frac{g^2}{6\pi}} \right] \quad (3.8)$$

and, for $\nu = 4$,

$$\square\phi = -\frac{\partial V}{\partial\phi} \quad (3.9)$$

The part of the effective action, S_F , associated with the fermionic determinant has the following form [16]

$$S_F = -g \int d^\nu x \int [D\phi] \mathcal{G}(x, x) \quad (3.10)$$

with the matrix element given by (3.5). Thus, for $\nu = 2$,

$$S_F = -\frac{g}{12\pi} \int d^2x \frac{\square\phi}{(\mu + g\phi)} \quad (3.11)$$

We note that, in the case of $\nu = 4$, the only effect of the fermions is (to this order of approximation) to renormalize the free parameters of the theory. We will see, in section V, that this situation is completely changed if we include a gap and bound states in the fermionic spectrum (see Fig. 4).

IV. THE SPHALERON SOLUTION

Based on the results of our previous work, we shall look for *sphaleron*-like solutions to the equations of motion obtained in the last section. A *sphaleron*-like solution corresponds to a field configuration that starts at the false vacuum ϕ_2 , almost reaches the true vacuum ϕ_1 , and returns to ϕ_2 , i.e., it looks like a droplet [1,2,6,9].

A. The $\nu = 2$ case

In this case, the equation of motion is (3.8). Defining $\varphi \equiv \phi + \mu/g$, using the thin-wall approximation [1] (i.e., considering a droplet whose radius is much larger than its wall thickness) and imposing the boundary conditions

$$\varphi_{sph}(x \rightarrow \pm\infty) \rightarrow \varphi_2 \quad (4.1)$$

$$\frac{d\varphi_{sph}}{dx}(x \rightarrow \pm\infty) \rightarrow 0 \quad (4.2)$$

we obtain

$$\varphi_{sph} = \varphi_2 - \phi_P [\tanh(\xi + \xi_0) - \tanh(\xi - \xi_0)] \quad (4.3)$$

where

$$\phi_P \equiv \left[\frac{(3\varphi_2^2 - \varphi_0^2)}{2} \right]^{1/2} \quad (4.4)$$

$$\xi \equiv g\phi_P(x - x_{c.m.}) \quad (4.5)$$

$$\xi_0 \equiv \frac{1}{2} \cosh^{-1} \left(\sqrt{\frac{2}{\frac{\varphi_0^2}{\varphi_2^2} - 1}} \right) \quad (4.6)$$

and where we have assumed $\mu/g \gg 1$ (neglecting corrections of order $O((g/\mu)^2)$), consistent with the conditions of validity of the gradient expansion, in order to obtain a closed form for the function that represents the sphaleron.

The parameter $x_{c.m.}$ reflects the translational invariance of the equation of motion and ξ_0 is related to what is usually called the radius of the sphaleron (see Fig. 2), being extremely important in the analysis of stability (see section VI).

B. The $\nu = 4$ case

In this case, the equation of motion is (3.9). Thus, assuming a solution with radial symmetry, we have, in the thin-wall approximation,

$$\varphi_{sph}(\tilde{\xi}) = \varphi_2 - \tilde{\phi}_P [\tanh(\tilde{\xi} + \tilde{\xi}_0) - \tanh(\tilde{\xi} - \tilde{\xi}_0)], \quad \tilde{\xi} \geq 0 \quad (4.7)$$

where we have imposed the following conditions

$$\tilde{\phi}_P \equiv \left(\frac{3\varphi_2^2 - \varphi_0^2}{2} \right)^{1/2} \quad (4.8)$$

$$\tilde{\xi} \equiv g\tilde{\phi}_P r \quad (4.9)$$

$$\tilde{\xi}_0 \equiv \frac{1}{2} \cosh^{-1} \left(\frac{2\varphi_2}{\sqrt{2(\varphi_0^2 - \varphi_2^2)}} \right) \quad (4.10)$$

and

$$\varphi_{sph}(r \rightarrow \infty) \rightarrow \varphi_2 \quad (4.11)$$

$$\frac{d\varphi_{sph}}{dr}(r \rightarrow \infty) \rightarrow 0 \quad (4.12)$$

Therefore, we have a true vacuum bubble, of radius $\tilde{\xi}_0$, centered at the origin (see Fig. 3). The situation is the same as we would have encountered in a purely bosonic system [1].

V. INCLUSION OF A GAP AND BOUND STATES

In the previous section, we have shown that a bubble-like configuration still satisfies the modified equations of motion (In fact, in the $\nu = 4$ case, we have no changes at all). These bubbles play the role of a background field in the determination of the fermionic spectrum. In fact, in some cases, instead of a simple continuum, we may have the presence of a gap and pairs of bound states [2,7] (we will consider, for simplicity, just one pair). Therefore, if we have a richer spectrum (see Fig. 4), the calculation of the fermionic determinant, performed in section III, should be reviewed to deal explicitly with the bound states. The calculation that led us to (3.11) is the result of a gradient expansion approximation for the total trace (2.3), where we sum over all continuum and bound states. However, as we are interested in an effect that depends crucially on the relative occupation of the bound states [2], we should consider a finite density of fermions, i.e., a partial trace. Thus, instead of summing over all fermionic momenta (the restriction of small momenta is over the bosonic fields only), we shall sum them only up to the (occupied) top bound state (see Fig. 4(c)).

Therefore, the complete effective action has the following form

$$\begin{aligned} S_{eff}[\phi] = & \int d^\nu x \left\{ \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - [V(\phi) - V(\phi_2)] \right\} - \\ & - \frac{g}{2} \int d^\nu x \int [D\phi] \mathcal{G}(x, x) - \end{aligned}$$

$$\begin{aligned}
& - n_+ g \int_0^T dt_1 \int d\vec{x} \int_0^T dt_2 \int d\vec{y} \int [D\phi] \psi_B^*(\vec{x}, t_1, [\phi_{sph}]) \times \\
& \times \mathcal{G}(x, y) \psi_B(\vec{y}, t_2, [\phi_{sph}])
\end{aligned} \tag{5.1}$$

where the first term is the bosonic contribution, the second term represents the “Dirac sea” (including antiparticle bound states) and the last one is due to the particle bound states. ψ_B is the wavefunction of a bound state and n_+ is its occupation number (“doping”).

The only term that remains to be calculated is the last one. Assuming that the occupation of the bound states will not affect in an appreciable way the form of the bubble (in the one-dimensional case, this is an exact result [2,7]), we may rewrite this term as (see Appendix B)

$$S_{\text{bound}} = n_+ g T \int d\vec{x} \rho(\vec{x}) \phi_{sph}(\vec{x}) \tag{5.2}$$

where $\rho(\vec{x}) = \psi_B^*(\vec{x}) \psi_B(\vec{x})$ is the normalized density of probability distribution of the bound charge. However, it is already known [5,7] that the charge associated with a bubble tends to concentrate on its surface in a gaussian-like way. For our purposes, we will assume that a delta-like distribution will be a reasonable approximation.

A. The $\nu = 2$ case

In this case, we may write the density ρ as

$$\rho = \frac{1}{2} [\delta(\xi - \xi_0) + \delta(\xi + \xi_0)] \tag{5.3}$$

The energy of the bubble, $E = -S_{\text{eff}}/T$, as a function of the radius ξ_0 , has the following form:

$$\begin{aligned}
E(\xi_0) \approx & \int_{-\infty}^{+\infty} \frac{d\xi}{g\phi_P} \left\{ \frac{1}{2} \left[1 + \frac{1}{12\pi} \left(\frac{g}{\mu} \right)^2 \right] (g\phi_P)^2 \left(\frac{d\phi_{sph}}{d\xi} \right)^2 + [V(\phi_{sph}) - V(\phi_2)] \right\} - \\
& - n_+ \tanh(2\xi_0)
\end{aligned} \tag{5.4}$$

where we have incorporated the contribution of the Dirac sea in the first term, by making use of the equation of motion. In fact, the contribution of the continuum is of order $O((g/\mu)^2)$

and may be neglected to this order of approximation. The only relevant contribution of fermions comes from the bound states.

B. The $\nu = 4$ case

In this case, we may write the density ρ as

$$\rho = \frac{(g\tilde{\phi}_P)^2}{4\pi\tilde{\xi}_0^2} \delta(\tilde{\xi} - \tilde{\xi}_0) \quad (5.5)$$

The energy of the bubble as a function of the radius $\tilde{\xi}_0$ has the following form:

$$E(\tilde{\xi}_0) \approx \int_0^\infty 4\pi \frac{\tilde{\xi}^2}{(g\tilde{\phi}_P)^2} \frac{d\tilde{\xi}}{(g\tilde{\phi}_P)} \left\{ \frac{1}{2}(g\tilde{\phi}_P)^2 \left(\frac{d\phi_{sph}}{d\tilde{\xi}} \right)^2 + [V(\phi_{sph}) - V(\phi_2)] \right\} - n_+ \tanh(2\tilde{\xi}_0) \quad (5.6)$$

VI. STABILITY OF THE BUBBLES

To analyze the stability of the bubble-like solutions obtained in section IV, we shall study the behaviour of the the energy of the bubble as a function of the bubble radius ξ_0 , now considered as a dynamical variable, s . The results, for the cases $\nu = 2$ and $\nu = 4$ obtained in last section, are plotted in Fig. 5 and Fig. 6, respectively. Our results for the $\nu = 2$ case should be compared with the exact results, previously obtained by using inverse scattering methods [2] (see Figs. 5(b) and 5(c)). In this way, we may control our approximations. In fact, the observation of Fig.5 shows that our approximation preserves the “quantum stabilization” brought about by fermions. Nevertheless, we find a quantitative difference between the approximate and the exact results due, mainly, to our naive delta-like approximation for the fermionic density. In order to improve on this approximation, one may use a gaussian-like pattern or even the exact fermionic density in the presence of the sphaleron. However, in doing so, one can no longer use the simple analytic form of the previous section.

VII. CONCLUSIONS AND COMMENTS

The aim of this work was to understand the role played by fermions in the decay of metastable states of a scalar field via bubble nucleation. Our basic approximation was a gradient expansion for the fermionic determinant that proved to work well in a comparison with exact results previously obtained for one-dimensional system. From our results, it is clear that the effects of fermions in an arbitrary number of space dimensions are due, almost exclusively, to the relative occupation of bound states. The states of the continuum contribute only to order $O((g/\mu)^2)$, and may be neglected.(This is in agreement with results obtained for an analogous problem in QCD [15].). For $\nu = 2$ and $\nu = 4$, the new charged bubbles have the same functional form as the purely bosonic ones, except for a reparametrization, and have their stability drastically modified by the fermions: besides unstable bubbles, we find metastable bubbles as a result of a “quantum stabilization” brought about by the fermions. For $\nu = 2$, this result was obtained exactly in Ref. [2]. This striking feature may, in principle, be measured in some realistic systems. The two-dimensional case finds a natural application in the physics of linearly conducting polymers [5–7]. The results for the four-dimensional case may have important consequences for the physics of Baryogenesis and in some problems in Optics [10,11]

From the results for the energy of the bubbles as a function of their radii, it is possible to calculate decay rates for the metastable states of the scalar field as explicit functions of time. This may be implemented by using the formalism presented in [1] and [2] and allows for a non-equilibrium description of the decay process for both $\nu = 2$ and $\nu = 4$ cases.

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APPENDIX A: GRADIENT EXPANSION OF THE FERMION DENSITY

The complete derivation of identity (3.3) can be found in ref. [12]. For completeness, we present here the main steps of its demonstration.

The density of fermions at a given point x_0 , in a background field $M(x)$, has the following form

$$\rho(x_0) \equiv \mathcal{G}(x_0, x_0) \equiv Sp \langle x_0 | \frac{1}{i\gamma^\mu \partial_\mu + M(\hat{x})} | x_0 \rangle \quad (\text{A1})$$

where $|x_0\rangle$ is a position eigenstate with eigenvalue x_0 , and Sp is the trace over spin degrees of freedom.

In momentum representation, this expression takes the form

$$\rho(x_0) = Sp \int \frac{d^\nu p}{(2\pi)^\nu} e^{ix_0^\mu p_\mu} \frac{1}{\gamma^\mu p_\mu + M(\hat{x})} e^{-ix_0^\mu p_\mu} \quad (\text{A2})$$

In order to transfer the x_0 dependence to $M(\hat{x})$, we make a unitary transformation that eliminates the exponentials from the last expression:

$$\rho(x_0) = Sp \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{\gamma^\mu p_\mu + M(\hat{x} + x_0)} \quad (\text{A3})$$

Note that x_0 in $M(\hat{x} + x_0)$ is a *c-number, not an operator*.

We can now expand $M(\hat{x} + x_0)$ around x_0 :

$$M(\hat{x} + x_0) = M(x_0) + \Delta M(x_0, \hat{x}) \quad (\text{A4})$$

where

$$\Delta M(x_0, \hat{x}) = \partial_\mu M(x_0) \hat{x}^\mu + \frac{1}{2} \partial_\mu \partial_\nu M(x_0) \hat{x}^\mu \hat{x}^\nu + \dots \quad (\text{A5})$$

Using (A4) in (A3) and considering that, in momentum space, $\hat{x}_\mu = \frac{1}{i} \frac{\partial}{\partial p_\mu}$, we have

$$\rho(x_0) = Sp \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{\gamma^\mu p_\mu + M(x_0) + \Delta M(x_0, \frac{1}{i} \frac{\partial}{\partial p_\mu})} \quad (\text{A6})$$

We can factor out $\frac{1}{\gamma^\mu p_\mu + M(x_0)}$ obtaining:

$$\rho(x_0) = Sp \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{\gamma^\mu p_\mu + M(x_0)} \left[1 + \Delta M(x_0, \frac{1}{i} \frac{\partial}{\partial p_\mu}) \times \frac{1}{\gamma^\mu p_\mu + M(x_0)} \right]^{-1} \quad (\text{A7})$$

The idea is to expand the last parenthesis in powers of $\Delta M/(\gamma^\mu p_\mu + M)$. This means that we consider smooth backgrounds $M(x_0)$, so that $|\partial_\mu M(x_0)| < |M(x_0)|$. In momentum space, this condition implies $|k_\mu \tilde{M}(k)| < |\tilde{M}(k)|$, where $k < 1$ is the momentum transferred from the fermions to the boson background. With this expansion we finally obtain:

$$\rho(x_0) = Sp \int \frac{d^\nu p}{(2\pi)^\nu} \frac{1}{\gamma^\mu p_\mu + M(x_0)} \sum_{n=0}^{\infty} (-1)^n \left(\Delta M(x_0, \frac{1}{i} \frac{\partial}{\partial p_\mu}) \times \frac{1}{\gamma^\mu p_\mu + M(x_0)} \right)^n \quad (\text{A8})$$

APPENDIX B: ENERGY OF BOUND STATES

In this appendix we derive equation (5.2).

From (5.1) we have

$$S_{\text{bound}} = -n_+ g \int dt_1 \int d\vec{x} \int dt_2 \int d\vec{y} \int [D\phi] \times \psi_B^\dagger(x, t_1, [\phi_{\text{sph}}]) \mathcal{G}(x, y) \psi_B(y, t_2, [\phi_{\text{sph}}]) \quad (\text{B1})$$

where ψ_B is the bound-state wave function in the presence of a sphaleron, and can be written in the following form

$$\psi_B(\vec{x}, t, [\phi_{\text{sph}}]) = \psi_B(\vec{x}) e^{iE_B t} \quad (\text{B2})$$

where E_B is the energy of the bound state.

The Green function in (B1) takes the form

$$\mathcal{G}(x, y) = \sum_n \psi_n^\dagger(\vec{y}) \psi_n(\vec{x}) e^{-iE_n(t_2 - t_1)} \quad (\text{B3})$$

where $E_n < \epsilon_F$ and

$$(i\gamma^\mu \partial_\mu + \mu + \phi) \psi_n(\vec{x}) = E_n \psi_n(\vec{x}) \quad (\text{B4})$$

The index n may be discrete or continuous, depending on the part of the spectrum we are considering. The sum must be done up to the Fermi energy ϵ_F .

Using (B3) in (B1), we obtain

$$S_{\text{bound}} = -n_+ g \int d\vec{x} \int d\vec{y} \int dt_1 \int dt_2 \int [D\phi] \sum_{n < \epsilon_F} \times \\ \times \psi_B^\dagger(\vec{x}) \psi_n(\vec{x}) \psi_n^\dagger(\vec{y}) \psi_B(\vec{y}) e^{-i(E_n - E_B)(t_2 - t_1)} \quad (\text{B5})$$

The difficulty in evaluating this expression resides in the ϕ -dependence of ψ_n . However, in the semiclassical context we consider fields “near” the sphaleron configuration. We also assume that the occupation of the bound states will not affect essentially the form of the bubble (this is an exact result in one spatial dimension [2]). Considering static fluctuations around the sphaleron, we can interchange the order of evaluation of the time and the ϕ integrals and, using the formula

$$\int dt_1 \int dt_2 e^{-i(E_n - E_B)(t_2 - t_1)} = T \delta(E_n - E_B)$$

we obtain

$$S_{\text{bound}} = -n_+ g T \int d\vec{x} \int d\vec{y} \int [D\phi] \psi_B^\dagger(\vec{x}) \psi_B(\vec{x}) \psi_B^\dagger(\vec{y}) \psi_B(\vec{y}) \quad (\text{B6})$$

provided that $\epsilon_F > E_B$.

Note that the $\delta(E_n - E_B)$ fixes the ϕ -dependence of ψ_n , so that we can integrate in ϕ and then evaluate the expression in the sphaleron configuration. This procedure leads to

$$S_{\text{bound}} = -n_+ g T \int d\vec{x} \int d\vec{y} \phi_{\text{sph}}(\vec{x}) \psi_B^\dagger(\vec{x}) \psi_B(\vec{x}) \psi_B^\dagger(\vec{y}) \psi_B(\vec{y}) \quad (\text{B7})$$

In this form, the \vec{x} and \vec{y} integrations decouple and, using orthonormality of the wave functions, we finally arrive at

$$S_{\text{bound}} = -n_+ g T \int d\vec{x} \phi_{\text{sph}}(\vec{x}) \psi_B^\dagger(\vec{x}) \psi_B(\vec{x}) \\ = -n_+ g T \int d\vec{x} \rho(\vec{x}) \phi_{\text{sph}}(\vec{x}) \quad (\text{B8})$$

which is precisely equation (5.2).

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[16] Here we used the trick of functionally differentiating and integrating the effective action, i.e., we used the relation $\frac{\delta}{\delta M} \text{tr}[\ln(i\gamma^\mu \partial_\mu + M(x))] = \mathcal{G}$, where \mathcal{G} is the Green function approximated by the gradient expansion.

Figure Captions:

Figure 1: Form of the potential $V(\phi)$.

Figure 2: The “sphaleron” solution for $\nu = 2$.

Figure 3: The “sphaleron” solution for $\nu = 4$.

Figure 4: (a) Continuum (gapless) fermionic spectrum. (b) Fermionic spectrum in the presence of a constant background. (c) Fermionic spectrum in the presence of a bubble-like background. Ω is a cutoff (in polymer applications, it corresponds to the bandwidth)

Figure 5: Energy of the bubble as a function of the bubble radius for $\nu = 2$. (a) purely bosonic. (b) zoom of the interesting region for the exact solution (including bound states). (c) zoom of the interesting region for the approximate solution (including bound states).

Figure 6: Energy of the bubble as a function of the bubble radius for $\nu = 4$. (a) purely bosonic. (b) zoom of the interesting region for the approximate solution (including bound states).

Figure 1

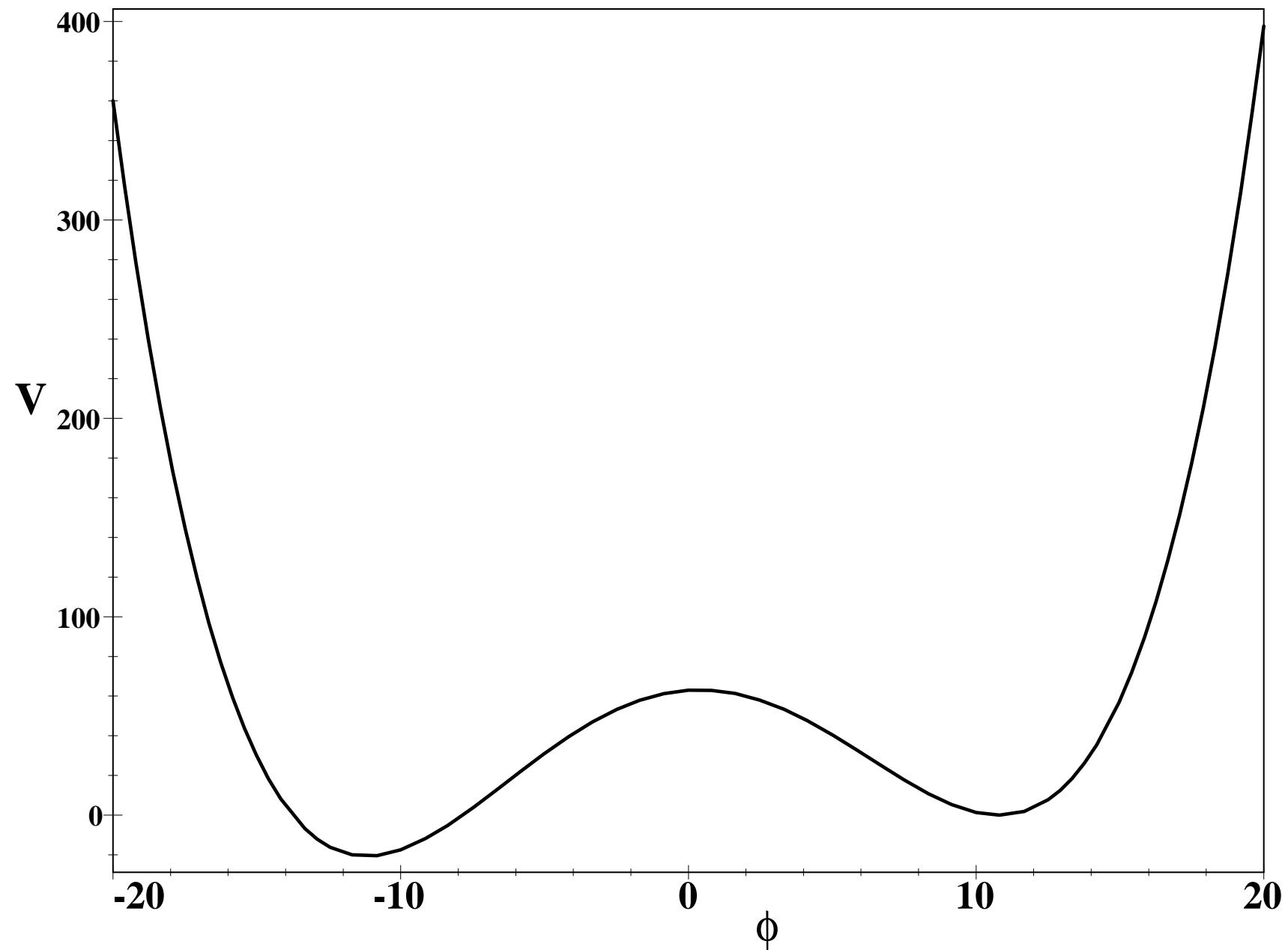


Figure 2

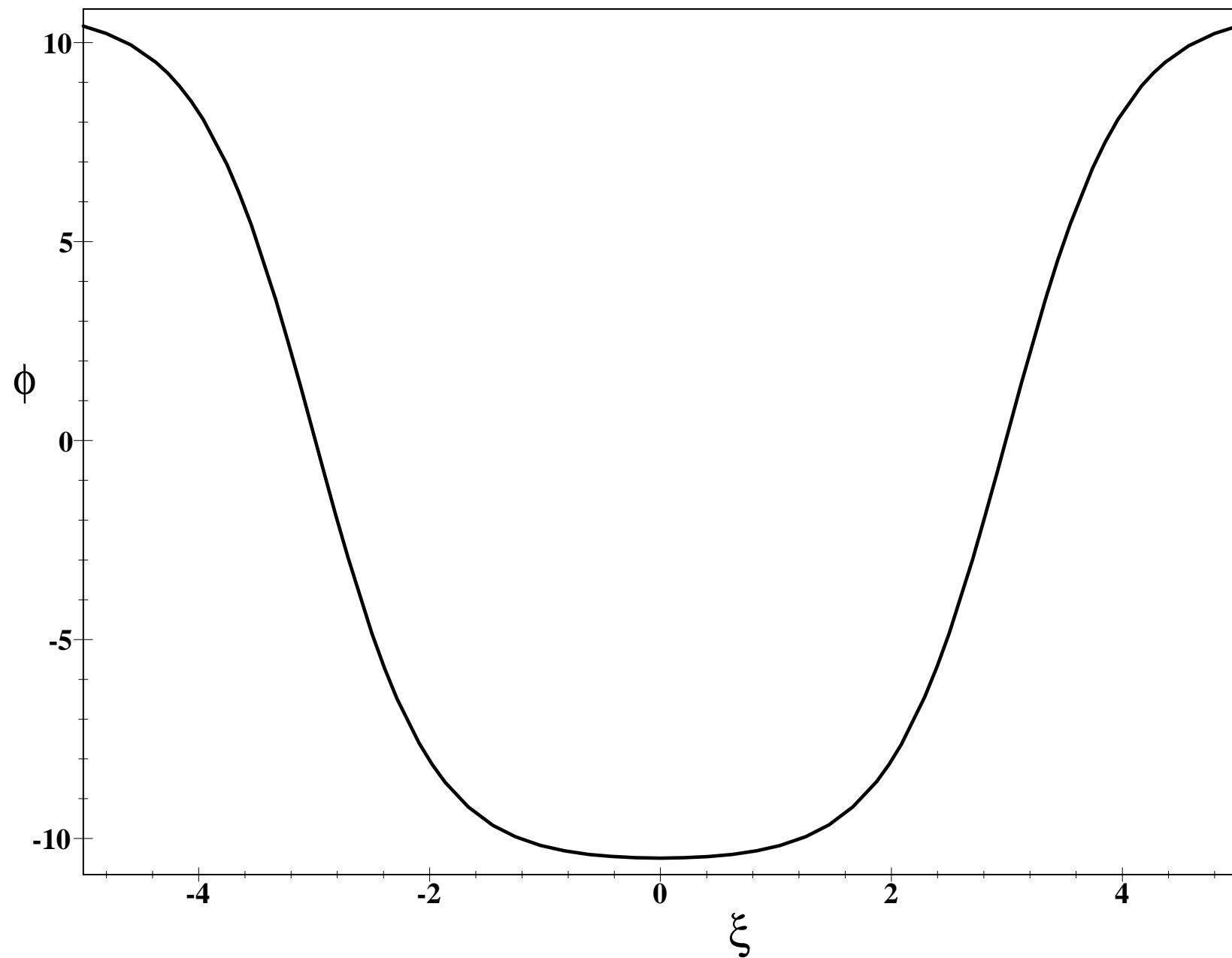


Figure 3

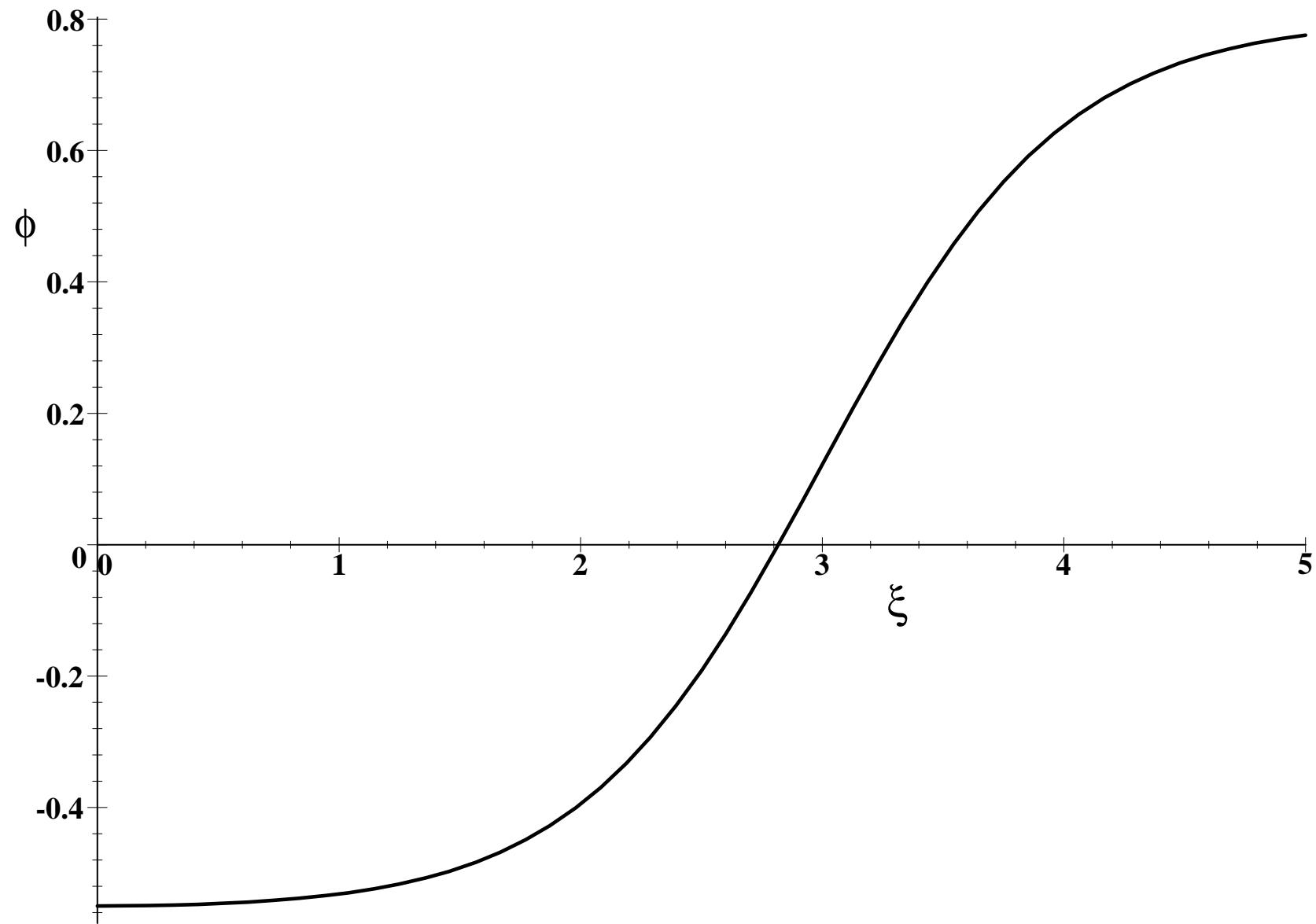


Figure 5(a)-bosonic

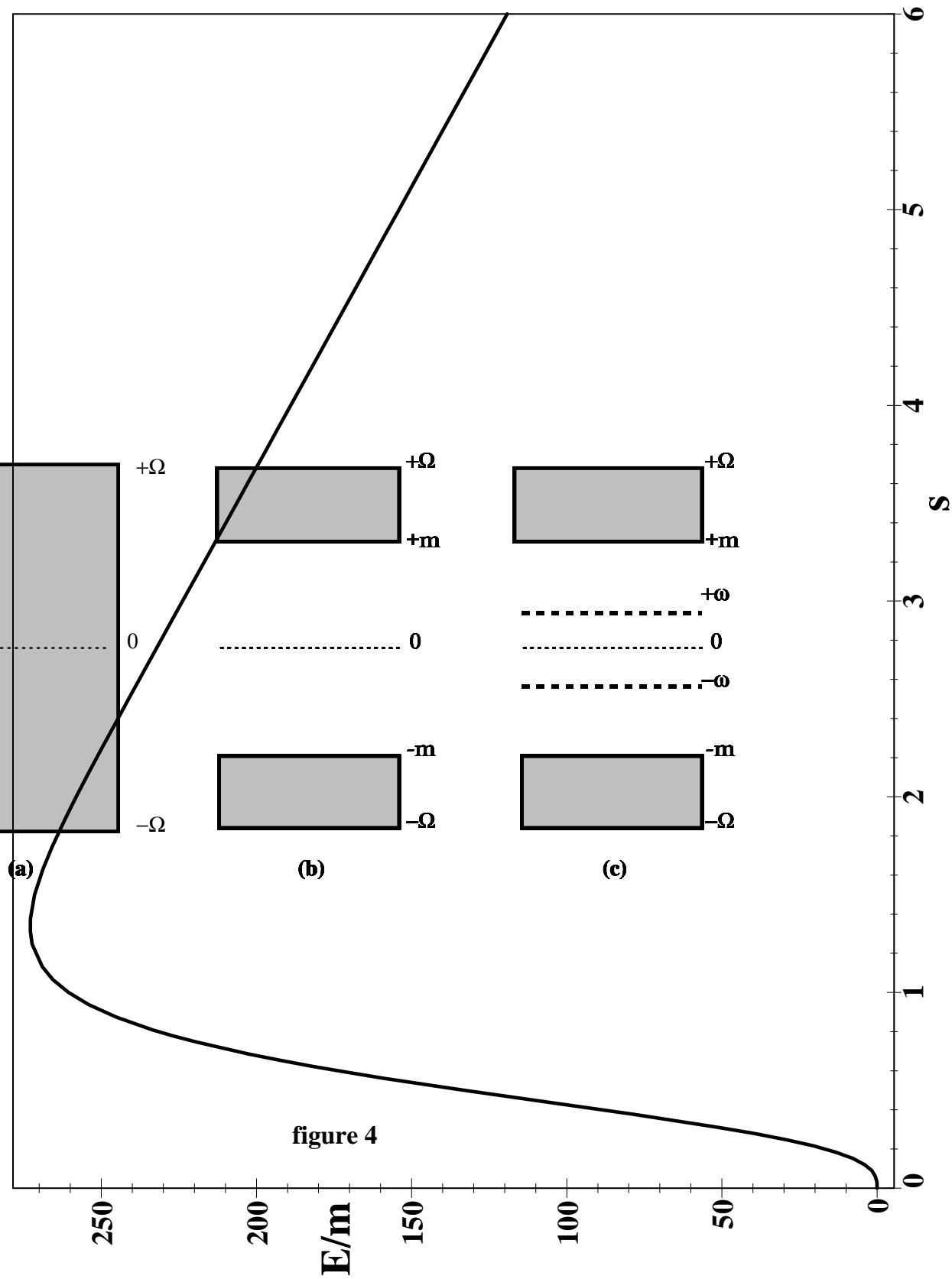


Figure 5(b)-exact

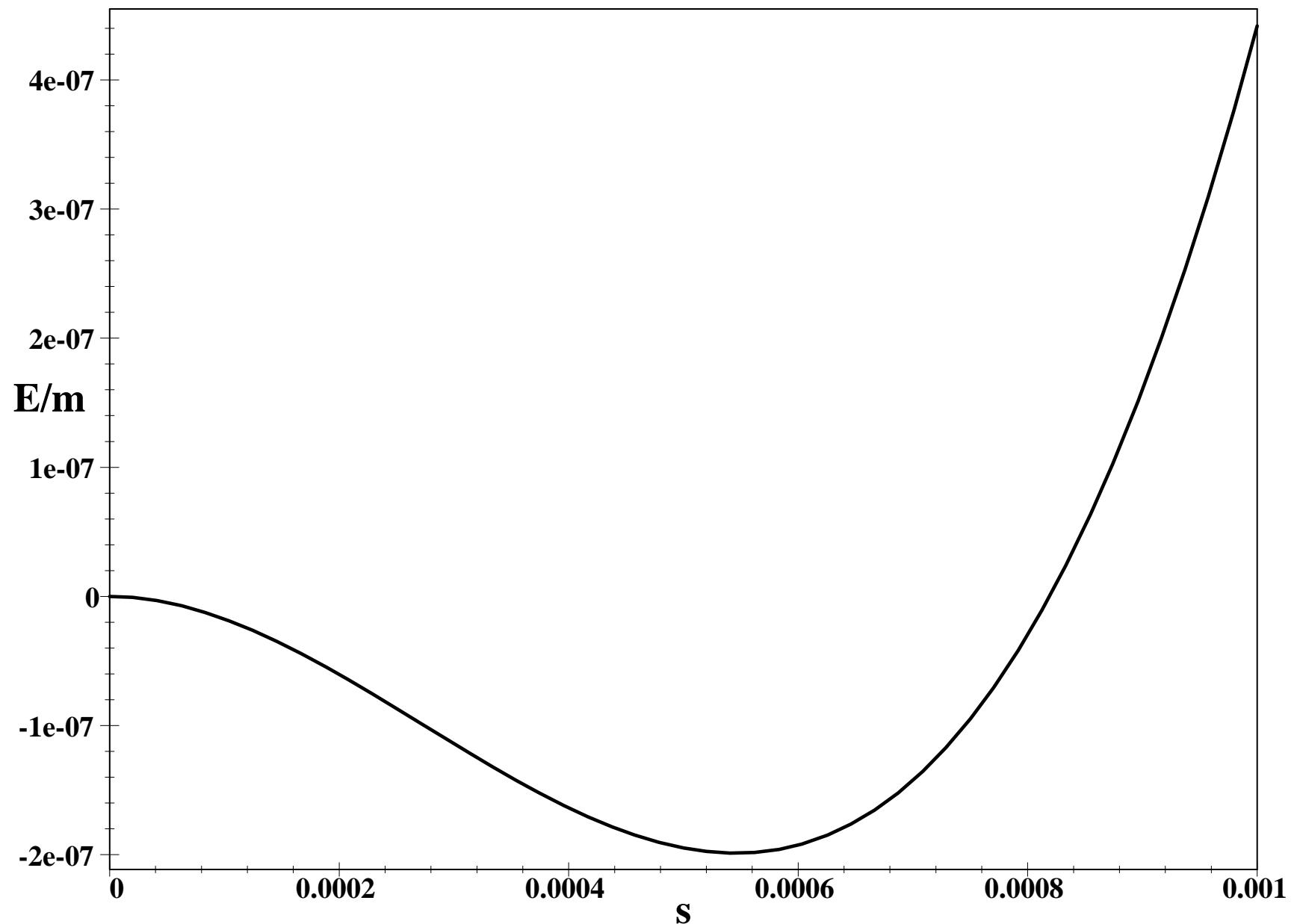


Figure 5(c)-approximate

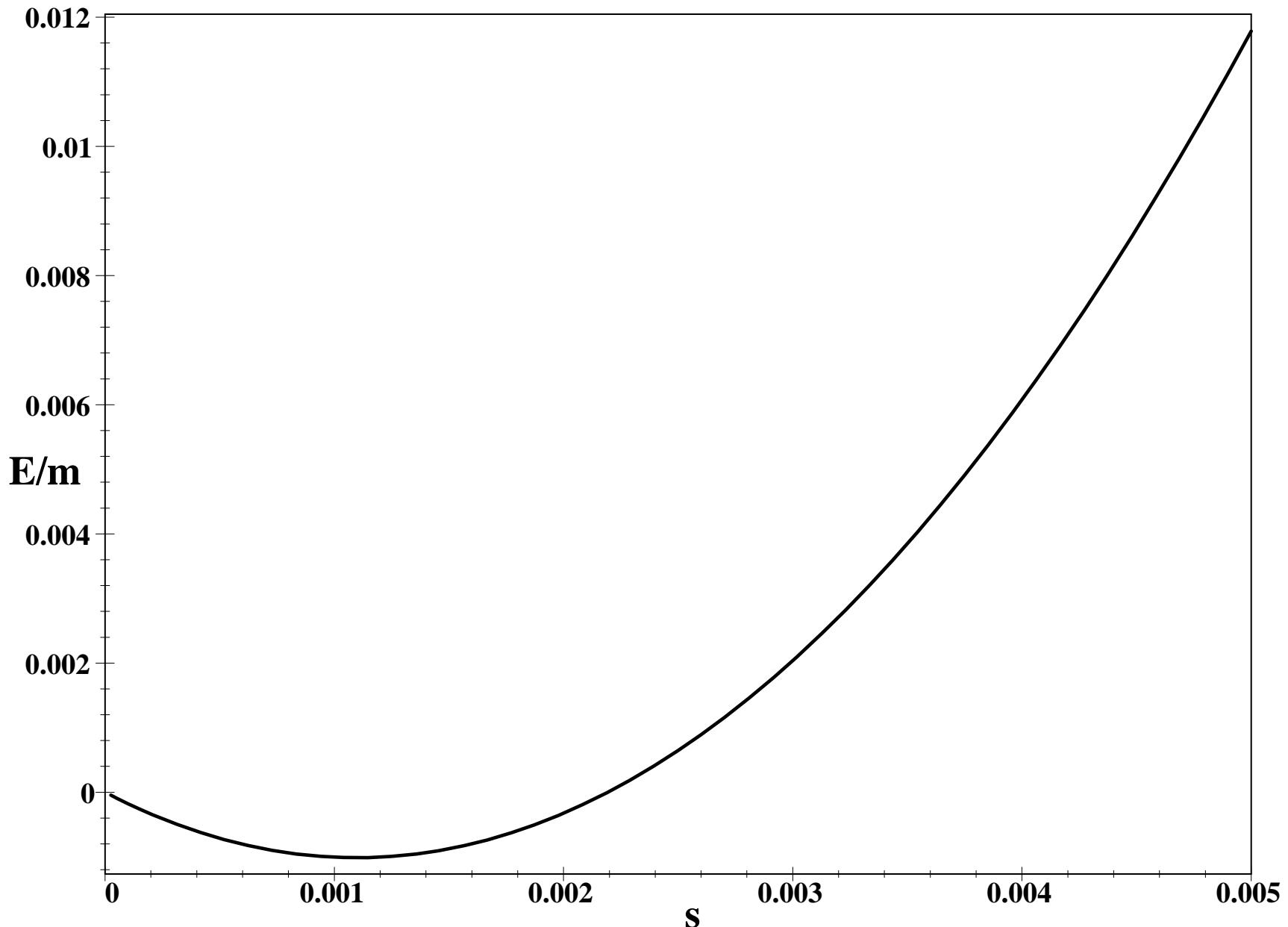


Figure 6(a)

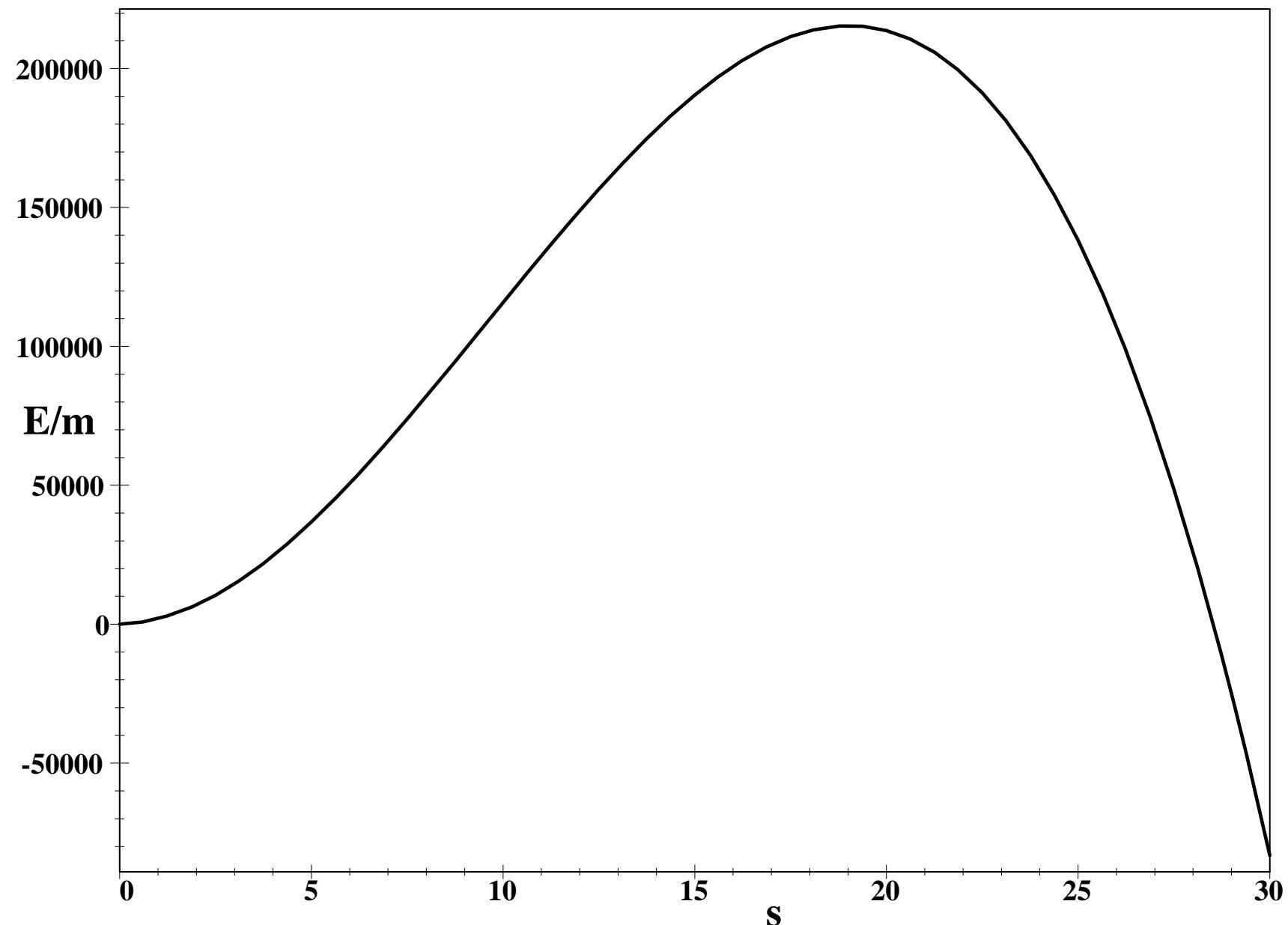


Figure 6(b)

